FP1 Specimen

1) $f(x) = x^3 - 3x^2 + 5x - 4$

a) $f'(x) = 3x^2 - 6x + 5$

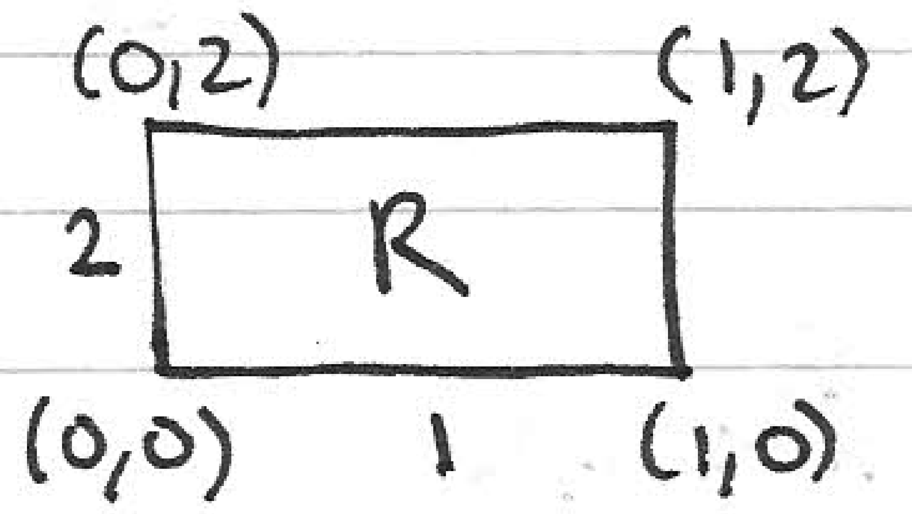
$x_0 = 1.4 \qquad x_1 = 1.4 - \dfrac{f(1.4)}{f'(1.4)} = \underline{1.455 \text{ (3dp)}}$

2) $AR = \begin{pmatrix} a & 4 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & a & a+8 & 8 \\ 0 & -1 & 1 & 2 \end{pmatrix}$

$(0,0)\,;\,(a,-1)\,;\,(a+8,1)\,;\,(8,2)$

b) $\det A = a + 4$

c)



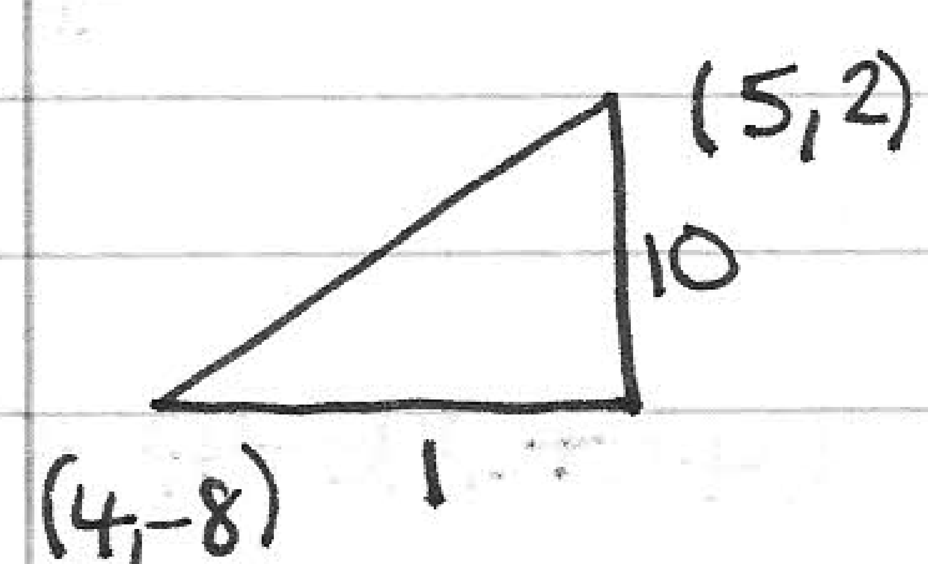
Area $R = 2$ Area Image $= 18$

$\Rightarrow \det A = 9 \Rightarrow 9 = a + 4 \Rightarrow \underline{a = 5}$

3) $R^2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ rotation $90°$ clockwise about origin.

c) $R =$ rotation $45°$ clockwise about origin.

4) $f(x) = 2^x - 6x \qquad f(4) = -8 \qquad f(5) = 2$



$m = \dfrac{10}{1} \qquad y - 2 = 10(x - 5)$

$y = 0 \Rightarrow -\frac{1}{5} = x - 5 \Rightarrow \underline{x = 4\frac{4}{5}}$

5) $\sum r^2 - \sum r - \sum 1 = \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - n$

$= \frac{1}{6}n\left[(n+1)(2n+1) - 3(n+1) - 6\right] = \frac{1}{6}n\left[2n^2 + 3n + 1 - 3n - 3 - 6\right]$

$= \frac{1}{6}n(2n^2-8) = \frac{1}{3}n(n^2-4) = \frac{1}{3}n(n+2)(n-2)$ #

b) $\sum_{10}^{40} r^2-r-1 = \frac{1}{3}(40)(42)(38) - \frac{1}{3}(9)(11)(7)$

$= \underline{21049}$

6) $z = -3+4i \quad |z| = \sqrt{3^2+4^2} = 5$

b) $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4}{-3} \quad \theta = \tan^{-1}\left(\frac{4}{-3}\right) = -0.927$

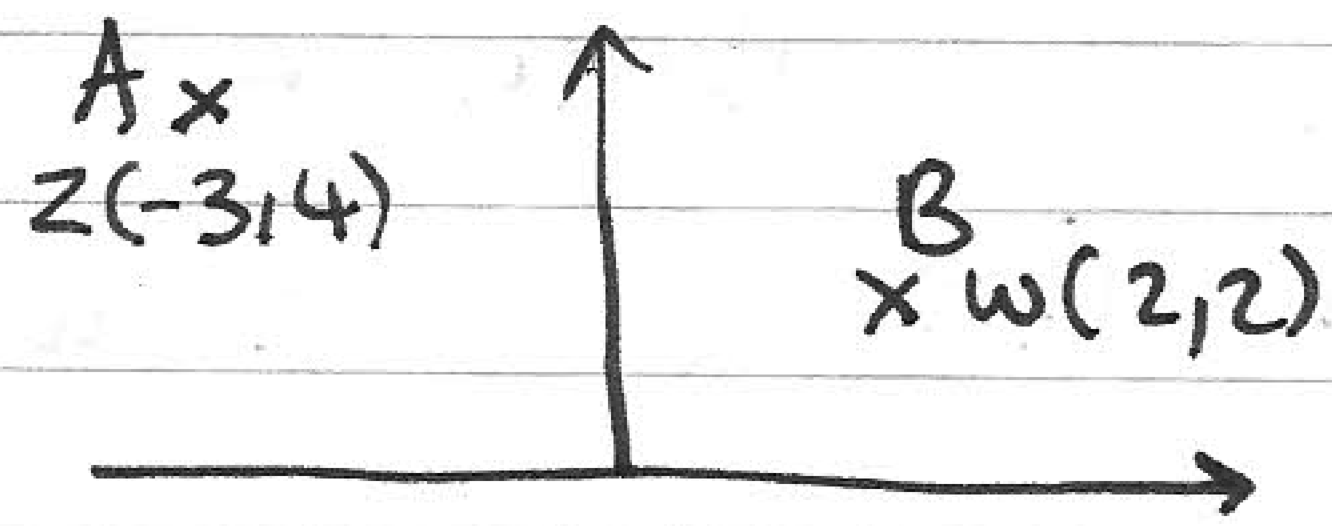
$+\pi$

$\arg(z) = 2.21$

c) $w = \frac{(14+2i)\times(-3-4i)}{(-3+4i)\times(-3-4i)} = \frac{42-8i^2+56i-6i}{9-16i^2} = \frac{50+50i}{25}$

$\underline{w = 2+2i}$

d)



7) $y^2 = 4ax \quad (4t^2, 8t) \Rightarrow 64t^2 = 4a(4t^2)$

$\Rightarrow 64t^2 = a \times 16t^2 \Rightarrow \underline{a = 4}$

b) $\frac{d}{dx}y^2 = \frac{d}{dx}16x \Rightarrow 2y\frac{dy}{dx} = 16 \Rightarrow \frac{dy}{dx} = \frac{8}{y} = \frac{8}{8t} \Rightarrow m_t = \frac{1}{t}$

$y - 8t = \frac{1}{t}(x - 4t^2) \Rightarrow yt - 8t^2 = x - 4t^2 \Rightarrow yt = x + 4t^2$ #

c) directrix $x + 4 = 0 \quad (-4, 15)$

$15t = -4 + 4t^2 \Rightarrow 4t^2 - 15t - 4 = 0 \quad (4t+1)(t-4) = 0$

$t = -\frac{1}{4} \quad t = 4$

$t = 4 \quad (64, 32) \quad t = -\frac{1}{4} \left(\frac{1}{4}, -2\right)$

8) $f(x) = 2x^3 - 5x^2 + px - 5$

a) $1-2i \Rightarrow$ other solution is $1+2i$ $\quad \alpha+\beta = 2$

$\alpha\beta = 1 - 4i^2 = 5$

b) $(x^2 - 2x + 5)(2x - 1) = 0 \Rightarrow x = 1-2i, 1+2i, \frac{1}{2}$

c) $10x + 2x = 12x \equiv px \Rightarrow \underline{p = 12}$

9) $\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}^n = \begin{pmatrix} n+1 & n \\ -n & 1-n \end{pmatrix}$

$n=1 \quad \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}^1 = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \qquad n=1 \begin{pmatrix} 1+1 & 1 \\ -1 & 1-1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$ ✓

$n = k+1 = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}^{k+1} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}^k$

$= \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} k+1 & k \\ -k & 1-k \end{pmatrix} = \begin{pmatrix} 2k+2-k & 2k+1-k \\ -k-1 & -k \end{pmatrix} = \begin{pmatrix} k+2 & k+1 \\ -(k+1) & -k \end{pmatrix}$

#

true for $n=1$, $n=k+1$ if true for $n=k$

$\therefore$ by Induction true for all $n \in \mathbb{Z}^+$

b) $f(n) = 4^n + 6n - 1 \qquad n=1 \quad f(1) = 4+6-1 = 9 = 3\times 3$ ✓

$n = k+1 \Rightarrow f(k+1) = 4^{k+1} + 6(k+1) - 1 = 4\times 4^k + 6k + 5$

$f(k+1) - f(k) = 4\times 4^k + 6k + 5 - 4^k - 6k + 1 = 3\times 4^k + 6$

$\Rightarrow f(k+1) = 3[4^k + 2] + f(k)$

true for $n=1$, true for $n=k+1$ if true for $n=k$ $\therefore$ by Induction true for all $n \in \mathbb{Z}^+$